

Name: Mahyar Pirayesh

Date: Nov. 25th, 2023

Math 12 Honors ch4 Review Trigonometric Functions and Transformations 2023

1. By definition, what is an odd function? What is an even function? Given examples of even and odd functions: An even function has the property that $f(x) = f(-x)$, where a H.R. does not change the appearance of the graph. An odd function has the property that $f(-x) = -f(x)$ meaning that the graph will look the same after both H.R. and V.R. together.
2. Which of the following equations below are true. CIRCLE All that are true

$\sin \theta = \sin(-\theta)$ $\sin \theta = -\sin(-\theta)$ $\cos \theta = \cos(-\theta)$ $-\cos \theta = \cos(-\theta)$

$\tan \theta = -\tan(\theta)$ $\cot \theta = -\cot(-\theta)$ $\csc \theta = \csc(-\theta)$ $\sec \theta = -\sec(-\theta)$

$\sec \theta = \frac{1}{\cos \theta}$

If $\cos \theta$ is an even function, then $\sec$ must also.

3. For each of the following trigonometric functions find the following:
- Period and amplitude
 - Phase shift
 - All "X" intercepts from -3π to 3π , also provide the general formula for all "x" intercepts
 - Maximum and Minimum points

a) $y = \frac{2}{3} \sin \frac{(x - \frac{2\pi}{3})}{4} + 11$

b) $y = -\frac{3}{4} \cos \left(2x - \frac{7\pi}{3} \right) - \frac{2}{7} = -\frac{3}{4} \cos 2 \left(x - \frac{7\pi}{6} \right) - \frac{2}{7}$

c) $y = 4.3 \tan \frac{3x-8}{5} - 2.1 = 4.3 \tan \left[\frac{3}{5} \left(x - \frac{8}{3} \right) \right] - 2.1$

d) $y = 8.5 \sin \frac{(8-2x)}{3} + 1 = 8.5 \sin \left[-\frac{2}{3} (x-4) \right] + 1 = -8.5 \sin \left[\frac{2}{3} (x-4) \right] + 1$

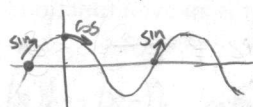
e) $y = -8.33 \tan \frac{3\pi(4-x)}{2} + 2 = -8.33 \tan \left(\frac{3\pi}{2} (-x+4) \right) + 2 = 8.33 \tan \left(\frac{3\pi}{2} (x-4) \right) + 2$

f) $y = -\frac{8}{3} \cos \frac{4\pi(8-3x)}{5} + 2.1 = -\frac{8}{3} \cos \left[\frac{12\pi}{5} (-x+8/3) \right] + 2.1 = -\frac{8}{3} \cos \left[\frac{12\pi}{5} (x - \frac{8}{3}) \right] + 2.1$

Work shown separately
(at end)

4. Given that $y = 3\cos\left(\frac{x-11}{4}\right) + 2$ is congruent to the function $y = 3\sin\left(\frac{(x-A)}{4}\right) + 2$, what is the smallest positive value of "A"? Show all your work.

Period: $\frac{2\pi}{\frac{1}{4}} = 8\pi$



we must subtract 2π

$$y = 3\sin\left(\frac{(x-11+2\pi)}{4}\right) + 2$$

$$A = 11 + 2\pi = 17.28$$

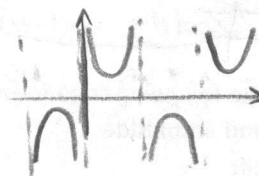
OR, we use $\cos\theta = \sin(\theta)$

5. For each of the following trigonometric functions find the following:

- Period
- Equation of all vertical asymptotes
- All "X" intercepts from -3π to 3π , also provide the general formula for all "x" intercepts
- Maximum and Minimum points

a) $y = -\frac{3}{2}\csc\left(x - \frac{3\pi}{4}\right) + 5$

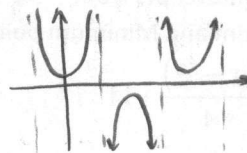
csc θ :



$$-\csc\theta = \cos(-\theta)$$

b) $y = -3.4\sec\pi\left(\frac{x-3}{4}\right) + 4$

sec θ :



$$\csc\theta = \csc(-\theta)$$

$$= -3.4\sec\left[\frac{\pi}{4}(x-3)\right] + 4$$

c) $y = -2.2\cot\frac{3\pi}{4}(2x-5) + 1$

$$= -2.2\cot\left[\frac{3\pi}{2}\left(x - \frac{5}{2}\right)\right] + 1$$

d) $y = 3.5\csc\frac{4\pi}{3}(3x-7) - 4$

$$y = 3.5\csc\left[4\pi\left(x - \frac{7}{3}\right)\right] - 4$$

e) $y = -\frac{2}{3}\cot\frac{3\pi}{2}(4-7x) + 3$

$$= -\frac{2}{3}\cot\left[-\frac{3\pi}{2}\left(x - \frac{4}{7}\right)\right] + 3 = \frac{2}{3}\cot\left(\frac{3\pi}{2}\left(x - \frac{4}{7}\right)\right) + 3$$

6. How many solutions are there in the interval from 0 to 2π ? Explain:

i) $3\sin 3\left(\theta - \frac{7\pi}{3}\right) + 2 = 0$

ii) $2\sin\frac{13}{2}\left(x - \frac{3\pi}{2}\right) + 1 = 0$

Period: $\frac{2\pi}{\frac{13}{2}} = \frac{4\pi}{13} \approx 0.9666$

Period: $\frac{2\pi}{3}$

graphically

$$2\sin\left[\frac{13}{2}\left(x - \frac{3\pi}{2}\right)\right] + 1 = 0$$

$$\theta = \sin^{-1}\left(-\frac{1}{2}\right) = \frac{13}{2}\left(x - \frac{3\pi}{2}\right)$$

$$x_1 = 4.63$$

$$x_2 = 5.276$$

4 periods + 1 period

- 5 periods + 1 period

12 total roots

7. For what values of "k" will there be no solutions? Explain:

i) $5\cos\frac{3\pi}{7}\left(\theta - \frac{\pi}{8}\right) + k = 0$

ii) $\frac{k}{2}\cos\left(\frac{4-\theta}{3}\right) + (k-4) = 0$

$$k > 5 \text{ or } k < -5$$

$$k-4 > \frac{1}{2}$$

$$\text{or } k-4 < -\frac{1}{2}$$

$$\frac{1}{2} - 4 > 0$$

$$k > 8$$

$$\frac{3k}{2} < 4 \Rightarrow k < \frac{8}{3}$$

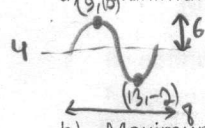
solving will give us 2 solutions. Then we have 3 periods: $2 \times 3 = 6$.



6 solution

8. Given the information about each trigonometric function, find a sinusoidal equation that describes it:

a) Maximum point at (9, 10) and the next minimum point at (13, -2)



$$y = 6 \sin\left(\frac{\pi}{4}(x-7)\right) + 4$$

$$p = \frac{2\pi}{B} \Rightarrow 8 = \frac{2\pi}{B} \Rightarrow B = \frac{\pi}{4}$$

b) Maximum point at $\left(\frac{2\pi}{3}, 11\right)$ and the next minimum point at $\left(\frac{9\pi}{2}, 3\right)$

$$D = \frac{11+3}{2} = 7$$

$$A = 11 - 7 = 4$$

$$B = \frac{2\pi}{23}$$

$$C =$$

$$p = 2\left(\frac{9\pi}{2} - \frac{2\pi}{3}\right) = 2\left(\frac{27\pi - 4\pi}{6}\right) = \frac{23\pi}{3}$$

$$p = \frac{2\pi}{B} \Rightarrow B = \frac{2\pi}{\frac{23\pi}{3}} = \frac{6}{23}$$

$$y = 4 \cos\left(\frac{6}{23}\left(x - \frac{2\pi}{3}\right)\right) + 7$$

c) Amplitude of 9, minimum point at (5.25, -2) and period of 10.

$$y = -9 \cos\left(\frac{\pi}{5}(x - 5.25)\right) + 11$$

9. High tide at 3am with water depth at 2.75m, low tide at 2:30pm with water depth at 1.75m.

a) Write an equation that describes the water depth throughout the day

$$y = \frac{1}{2} \cos\left(\frac{2\pi}{23}(x-3)\right) + 2.25$$

$$\text{Amplitude} = A = \frac{2.75 - 1.75}{2} = \frac{1}{2} \text{ m}$$

$$D = \frac{2.75 + 1.75}{2} = 2.25 \text{ m}$$

$$B = \frac{2\pi}{23}$$

$$C = +3$$

dis. between max & min 11.5 hr

Period: 23 hours

$$23 = \frac{2\pi}{B} \Rightarrow B = \frac{2\pi}{23}$$

b) What is the water depth at noon?

$$y = \frac{1}{2} \cos\left(\frac{2\pi}{23}(12-3)\right) + 2.25$$

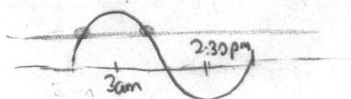
$$= 1.86 \text{ m}$$

c) In order to set sail, ocean depth must be at least 2.35 meters deep. At what times throughout the day will you be able to set sail?

$$\frac{1}{2} \cos\left(\frac{2\pi}{23}(x-3)\right) + 2.25 = 2.35$$

$$x_1 = 8.01$$

$$x_2 = -2.0129 \Rightarrow 20.987$$



Between 8:59 pm and 8 am

10. Suppose the distance between two planetary objects follow a sinusoidal function. In three years, the two planets will be at their closest at 4.5 AU (astronomical units - 1.496e+8 km). In 22 years, the two planets will be at their farthest at 10.5 AU. The cycle repeats itself.

a) Write an equation that describes the distances [AU] between the two planets as a function of the number of years

$$A = \text{amplitude} = \frac{10.5 - 4.5}{2} = 3 \text{ AU}$$

$$D = \frac{10.5 + 4.5}{2} = 7.5 \text{ AU}$$

$$B = \frac{\pi}{19}$$

$$C = 22$$

$$y = 3 \cos\left(\frac{\pi}{19}(x-22)\right) + 7.5$$

Period = 38 year

$$B = \frac{2\pi}{38} = \frac{\pi}{19}$$

b) How far will the two planets be from each other in 10 years?

$$y = 3 \cos\left(\frac{\pi}{19}(10-22)\right) + 7.5 = 6.29 \text{ AU}$$

c) To travel between the two planets, the distances between them must be within 6AU. So in the next 100 years, what percentage of that time is it conducive to interplanetary travel between the two planets?

$$6 = 3 \cos\left(\frac{\pi}{19}(x-22)\right) + 7.5$$

$$x_1 = 34.67$$

$$x_2 = 47.33$$

Between 0 and 9.33 years
Between 34 and 47 years

multiple cycles

$$34.67\%$$

$$3a) y = \frac{2}{3} \sin \frac{1}{4} \left(x - \frac{2\pi}{3} \right) + 11$$

$$\text{Period} = \frac{2\pi}{\frac{1}{4}} = \boxed{8\pi}$$

$$\text{Amplitude} = \boxed{\frac{2}{3}}$$

$$\text{Phase shift: } \boxed{\frac{2\pi}{3} \text{ right}}$$

$$\text{X-intercepts: } \emptyset$$

$$\text{Max: } \left(\frac{8\pi}{3} + 8\pi n, \frac{35}{3} \right); n \in \mathbb{Z}$$

$$\text{Min: } \left(-\frac{4}{3}\pi + 8\pi n, \frac{31}{3} \right); n \in \mathbb{Z}$$

$$b) y = -\frac{3}{4} \cos \left(2x - \frac{7\pi}{3} \right) - \frac{2}{7} \Rightarrow y = -\frac{3}{4} \cos 2 \left(x - \frac{7}{6}\pi \right) - \frac{2}{7}$$

$$\text{Period: } \frac{2\pi}{2} = \boxed{\pi}$$

$$\text{Amplitude: } \boxed{\frac{3}{4}}$$

$$\text{Phase shift: } \frac{7\pi}{3} \text{ right}$$

$$\text{X-intercepts: } -\frac{3}{4} \cos 2 \left(x - \frac{7}{6}\pi \right) - \frac{2}{7} = 0$$

$$\cos \theta = \frac{2}{7} \cdot \frac{4}{3}$$

$$\theta = \cos^{-1} \left(\frac{8}{21} \right) = 1.06^R \quad \theta_2 = -\theta_1 = -1.06^R$$

$$2 \left(x - \frac{7}{6}\pi \right) = 1.06^R$$

$$x = 4.65^R$$

$$2 \left(x - \frac{7}{6}\pi \right) = -1.06^R$$

$$x = 2.68^R$$

$$(4.65^R + n\pi, 0), (2.68^R + n\pi, 0); n \in \mathbb{Z}$$

$$(-7.92^R, 0), (-4.78^R, 0), (-1.64^R, 0), (1.50^R, 0), (4.65^R, 0), (7.79^R, 0)$$

$$(-6.74^R, 0), (-3.60^R, 0), (-0.46^R, 0), (2.68^R, 0), (5.83^R, 0), (8.97^R, 0)$$

c) Period: $\frac{\pi}{\frac{3}{5}} = \frac{5\pi}{3}$

Amplitude: 4.3

Phase shift: $\frac{8}{3}^R$ right

Max/min: N/A

X-intercepts:

$$4.3 \tan \left[\underbrace{\frac{3}{5} \left(x - \frac{8}{3} \right)}_{=\theta} \right] - 2.1 = 0$$

$$\tan \theta = \frac{2.1}{4.3}$$

$$\theta = \frac{3}{5} \left(x - \frac{8}{3} \right) = \tan^{-1} \left(\frac{2.1}{4.3} \right)$$

$x_1 = 3.42^R$ NOTE THAT only one answer exists.

$$(x, y) = \left(3.42^R + \frac{5\pi}{3}n, 0 \right); n \in \mathbb{Z}$$

$$(x, y) = (-7.05^R, 0), (-1.81^R, 0), (3.42^R, 0), (8.66^R, 0)$$

d) Period: $\frac{2\pi}{\frac{2}{3}} = 3\pi$

Roots: $-8.5 \sin \left(\underbrace{\frac{2}{3}(x-4)}_{=\theta} \right) + 1 = 0$

Amplitude: 8.5

$$\sin \theta = \frac{1}{8.5}$$

Phase change: 4 Right

$$\theta = \frac{2}{3}(x-4) = 0.1179^R \text{ or } 3.024^R$$

Max: $\left(4 - \frac{3}{4}\pi + 3n\pi, 9.5 \right); n \in \mathbb{Z}$

$$x = 4.18^R$$

$$x = 8.535$$

$$(x, y) = (4.18^R + 3n\pi, 0), (8.535^R + 3n\pi, 0); n \in \mathbb{Z}$$

Min: $\left(4 + \frac{3}{4}\pi + 3n\pi, -7.5 \right); n \in \mathbb{Z}$

$$(x, y) = (4.18^R, 0), (8.536^R, 0), (-5.25^R, 0), (-0.88^R, 0)$$

e) Period: $\frac{\pi}{\frac{3\pi}{2}} = \frac{2}{3}^R$

Phase shift: 4^R right

Amplitude: 8.33^R

Max/min: N/A

X-intercept: $8.33 \tan\left(\underbrace{\frac{3\pi}{2}(x-4)}_{=\theta}\right) + 2 = 0$

$$\tan \theta = \frac{-2}{8.33}$$

$$\theta = \frac{3\pi}{2}(x-4) = \tan^{-1}\left(\frac{-2}{8.33}\right)$$

$$x = 3.95^R$$

$$(x, y) = \left(3.95^R + \frac{2}{3}n, 0\right); n \in \mathbb{Z}$$

$$(x, y) = (3.95^R, 0), (3.28^R, 0), (2.62^R, 0), (1.95^R, 0) \dots$$

too many to list

f) Period: $\frac{2\pi}{\frac{12\pi}{5}} = \frac{5}{6}$

Phase shift: $\frac{8}{3}$ right

Amplitude: $\frac{8}{3}$

Max: $\left(\frac{21}{12} + \frac{5}{6}n, \frac{143}{30}\right)$

Mm: $\left(\frac{8}{3} + \frac{5}{6}n, -\frac{17}{30}\right)$

$n \in \mathbb{Z}$

X-int: $-\frac{8}{3} \cos\left[\underbrace{\frac{12\pi}{5}(x-\frac{8}{3})}_{=\theta}\right] + 2.1 = 0$

$$\cos \theta = 0.7875$$

$$\theta = \frac{12\pi}{5}(x-\frac{8}{3}) = 0.664 \text{ or } -0.664$$

$$x_1 = 2.75^R \quad x_2 = 2.579^R$$

$$(x, y) = \left(2.75^R + \frac{5}{6}n, 0\right), \left(2.579^R + \frac{5}{6}n, 0\right); n \in \mathbb{Z}$$

too many roots to list

Hilroy

5a) Period: 2π
V.A.:

$$0, \pi, 2\pi, 3\pi \rightarrow \frac{3\pi}{4}, \dots$$

$$\boxed{x = \frac{3\pi}{4} + \pi n, n \in \mathbb{R}}$$

Max/min: $\emptyset$

X-int:

$$-\frac{3}{2} \csc \left(\underbrace{x - \frac{3\pi}{4}}_{=\theta} \right) + 5 = 0$$

$$-\frac{3}{2} \left(\frac{1}{\sin \left(x - \frac{3\pi}{4} \right)} \right) + 5 = 0$$

$$\theta = x - \frac{3\pi}{4} = \sin^{-1} \left(\frac{1}{(-5)(-\frac{2}{3})} \right)$$

$$x_1 = 2.66$$

$$x_2 = 5.19$$

$$(x, y) = (2.66 + 2\pi n, 0), (5.19 + 2\pi n, 0); n \in \mathbb{Z}$$

$$(x, y) = (2.66, 0), (-3.62, 0), (8.94, 0)$$

$$(5.19, 0), (-1.09, 0), (-7.38, 0)$$

b) Period: $\frac{2\pi}{\frac{\pi}{4}} = 8^R$

$$\text{V.A.}: \frac{\pi}{4} \rightarrow \frac{\pi}{4} + 3 = 3^R$$

$$\boxed{x = 3^R + 8n^R; n \in \mathbb{Z}}$$

Max/min: $\emptyset$

$$\text{X-int: } -3.4 \sec \left[\underbrace{\frac{\pi}{4}(x-3)}_{=\theta} \right] + 4 = 0$$

$$\frac{1}{\cos \theta} = \frac{4}{-3.4} \Rightarrow \theta = \cos^{-1} \left(\frac{3.4}{4} \right)$$

$$x_1 = 3.7^R$$

$$x_2 = 2.29^R$$

$$(x, y) = (3.7^R + 8n^R, 0) / (2.29^R + 8n^R, 0); n \in \mathbb{Z}$$

$$c) \text{ Period: } \frac{\pi}{\frac{3\pi}{2}} = \frac{2}{3}^R$$

$$V.A.: 0 \rightarrow \frac{5}{2}^R$$

$$x = \frac{5}{2}^R + \frac{2}{3}^R n; n \in \mathbb{Z}$$

$$\text{Max/min} = \emptyset$$

$$\text{Roots: } -2.2 \cot \left[\underbrace{\frac{3\pi}{2} \left(x - \frac{5}{2} \right)}_{=\theta} \right] + 1 = 0$$

$$-2.2 \left(\frac{1}{\tan \theta} \right) + 1 = 0$$

$$\theta = \frac{3\pi}{2} \left(x - \frac{5}{2} \right) = \tan^{-1} \left(\frac{2.2}{1} \right)$$

$$x = 2.74^R$$

$$(x, y) = \left(2.74^R + \frac{2}{3}^R n, 0 \right); n \in \mathbb{Z}$$

$$d) \text{ Period: } \frac{2\pi}{4\pi} = \frac{1}{2}^R$$

$$V.A.: \frac{7}{3}^R$$

$$\text{Max/min} = \emptyset$$

$$\text{Roots: } 3.5 \csc \left[\underbrace{4\pi \left(x - \frac{7}{3} \right)}_{=\theta} \right] - 4 = 0$$

$$\theta = \sin^{-1} \left(\frac{3.5}{4} \right) = 4\pi \left(x - \frac{7}{3} \right)$$

$$x_1 = 2.42^R$$

$$x_2 = 2.499^R$$

$$(x, y) = \left(2.42^R + \frac{1}{2}^R n, 0 \right), \left(2.499^R + \frac{1}{2}^R n, 0 \right); n \in \mathbb{Z}$$

$$e) \text{ Period: } \frac{\pi}{2\pi} = \frac{2}{21}^R$$

$$V.A.: \frac{4}{7}^R + \frac{2}{21}^R n$$

$$\text{max/min} = \emptyset$$

$$\text{Roots: } \frac{2}{3} \cot \left(\underbrace{\frac{2\pi}{21} \left(x - \frac{4}{7} \right)}_{=\theta} \right) + 3$$

$$\theta = \frac{2\pi}{21} \left(x - \frac{4}{7} \right) = \tan^{-1} \left(\frac{2}{-3} \right) \Rightarrow x = 0.5648^R$$

$$(x, y) = \left(0.5648 + \frac{2}{21}^R n, 0 \right); n \in \mathbb{Z}$$